Short Papers

Relaxed Resilient Fuzzy Stabilization of Discrete-Time Takagi–Sugeno Systems via a Higher Order Time-Variant Balanced Matrix Method

Xiangpeng Xie, *Member, IEEE*, Cong Wei, Zhou Gu, *Member, IEEE*, and Kaibo Shi, *Member, IEEE*

***Abstract*—Resilient fuzzy stabilization is capable of providing much less conservative results than conventional fuzzy stabilization while the alert threshold condition should be always satisfied at each sampling instant. In order to make the alert threshold condition more easily to be guaranteed, the short paper employs the switching-type gain-scheduling control law so that the real-time information, which is specific to the current sampling instant, can be integrated into resilient fuzzy stabilization. More importantly, a new kind of time-variant balanced matrix is introduced for the first time for adjusting positive/negative terms of different monomials in a more flexible way. As a result, the conservatism of resilient fuzzy stabilization can be further reduced even if the alert threshold condition becomes more difficult to be violated. Finally, the advantage of the developed method is tested and validated via related comparisons on a benchmark example.**

***Index Terms*—Fuzzy control, gain-scheduling, nonlinear control, resilient stabilization.**

I. Introduction

In the past two decades, fuzzy control has been deeply investigated in order to deal with complex nonlinear systems, see recent results reported in [1]–[5] and the literature therein. Takagi–Sugeno (T–S) fuzzy systems [6] in especial have been applied to give exact expressions of different nonlinear dynamics within a required range of validity [7]. Therefore, the T–S fuzzy-model-based control synthesis of nonlinear systems can be studied by using the Lyapunov direct approach, e.g., [8]–[12]. It is important to consider the problem of fuzzy stabilization since it does play an important role in fuzzy control [13]. However, there still exist two main challenges for fuzzy stabilization to be addressed although they have been pointed out in the review paper [14] as follows: 1) how to alleviate the conservativeness of designing conditions via linear matrix inequalities (LMIs), and 2) how to provide the more economical fuzzy controller at the expense of a smaller implementation cost.

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Xiangpeng Xie and Cong Wei are with the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210023, China (e-mail: xiexiangpeng1953@163.com).

Zhou Gu is with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China (e-mail: gzh1808@163.com).

Kaibo Shi is with the School of Information Science and Engineering, Chengdu University, Chengdu 610106, China.

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As far as the challenge 1) is concerned, the conservatism of the parallel distributed compensation (PDC) scheme has been relaxed by various techniques, such as the set theory [15], local stability [16], [17], tensor product transformation [18], piecewise Lyapunov functions [19], [20], and so on. Especially, another fuzzy control law, i.e., nonparallel distributed compensation (non-PDC), has been proposed in [21], and thus, conventional fuzzy stabilization has got tremendous development [22]. Recently, much relaxed results have been obtained by means of increasing the order of gain matrices, such as multiple-parameterization method [23], multiple-steps method [24], homogeneous polynomials method [25] inspired by [26], delayed nonquadratic method [27], multi-instant homogenous polynomials method [28], [29], and multiple-sums argument method [30]. Nevertheless, it must be said there exists a compromise between challenge 1) (about conservatism) and challenge 2) (about complexity) of the conventional fuzzy stabilization, and thus, the computational burden in the previous literature may become too heavy to be implemented [31]. In other words, those two main challenges mentioned in [14] still remain in this field and are difficult to be well addressed under the framework of conventional fuzzy stabilization. More recently, the resilient fuzzy stabilization has been developed in [32] in which much better results can be obtained over conventional fuzzy stabilization. However, it should be noted that the rigorous alert threshold condition of resilient fuzzy stabilization needs to be satisfied at each sampling instant. Therefore, how to make the alert threshold condition more easily to be guaranteed has become an interesting topic of this direction.

In contrast to the recent result of [32], this short article aims at developing relaxed resilient fuzzy stabilization while the alert threshold condition becomes more difficult to be violated. To do this, the switching-type gain-scheduling control law is employed so that the real-time information that is specific to the current sampling instant can be integrated into the resilient fuzzy stabilization. More importantly, a new kind of time-variant balanced matrix is introduced for the first time for adjusting positive/negative terms of different monomials in a more flexible way. Indeed, the obtained result can be much less conservative than the recent result of [32] even if the alert threshold condition becomes more difficult to be violated. Finally, the advantage of the developed method will be tested and validated via related comparisons on a benchmark example.

Notations: Throughout this short article, our applied notations are the same as those cited literature, such as [31] and [32]. For instance, $\mathbb{Z}_+$ represents the set of positive integers. $d!$ means the factorial of any natural number d. $\mathrm{He}(U) = U + U^T$, where U represents one appropriately dimensional matrix.

II. Preliminaries

A. Discrete-Time T–S Fuzzy System

The discrete-time T–S fuzzy system consists of a group of IF-THEN rules [6]:

IF-THEN Rule l: IF $\varsigma_1(t)$ belongs to $\mathcal{M}_1^l, \cdots, \varsigma_p(t)$ belongs to $\mathcal{M}_p^l$, then

$$x(t+1) = A_l x(t) + B_l u(t), \quad l \in \{1, \ldots, r\} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_1}$ represents plant's state vector, $u(t) \in \mathbb{R}^{n_2}$ represents plant's input vector, and $\varsigma(t) = (\varsigma_1(t), \ldots, \varsigma_p(t))^T$ represents the plant's fuzzy premise variable vector. We have $n_1, n_2 \in \mathbb{Z}_+$.

Using the sector nonlinearity method [6], the overall T–S fuzzy system for (1) is represented as

$$\begin{aligned} x(t+1) &= \sum_{l=1}^{r} h_l(\varsigma(t))(A_l + B_l u(t)) \\ &= A_{\varsigma(t)} x(t) + B_{\varsigma(t)} u(t) \end{aligned} \tag{2}$$

where $h_l(\varsigma(t))$ represents the lth current-time normalized fuzzy weighting function (NFWF) satisfying the convex properties: $h_l(\varsigma(t)) \geq 0$ and $\sum_{l=1}^{r} h_l(\varsigma(t)) = 1$, and $A_{\varsigma(t)} = \sum_{l=1}^{r} h_l(\varsigma(t)) A_l$ and $B_{\varsigma(t)} = \sum_{l=1}^{r} h_l(\varsigma(t)) B_l$.

B. Homogeneous Matrix Polynomials

A number of needful preliminaries about homogeneous matrix polynomials given in the literature [26], [28] are reviewed here.

For $k_j \in \mathbb{Z}_+, j \in \{1, \ldots, r\},\ k = k_1 k_2 \cdots k_r$ represents the r-tuple, $\mathcal{K}(s)$ represents a group of r-tuples consist of all possible $k = k_1 k_2 \cdots k_r$ such that $\sum_{j=1}^{r} k_j = s$ and $s \in \mathbb{Z}_+$. $h_1(\varsigma(t))^{k_1} \cdots h_r(\varsigma(t))^{k_r}$ represents the monomial and matrix U_k represents the matrix-valued coefficient. Furthermore, $\pi(k)$ represents $\pi(k) = \prod_{j=1}^{r} (k_j!)$ and χ_j represents $0 \cdots \underbrace{1}_{j-\text{th}} \cdots 0$. Give two r-tuples k and k', we write $k - k' \geq 0$ if $k_j - k_j' \geq 0$ for all $j \in \{1, \ldots, r\}$. The operations of summation $(k + k')$ and subtraction $(k - k')$ are both operated componentwise.

In order to save some space, the following simplifications are employed with $\forall k \in \mathcal{K}(d), d \in \mathbb{Z}_+$:

$$\begin{cases} h_l(t) = h_l(\varsigma(t)),\ h(t) = (h_1, \ldots, h_r)^T \\ h(t)^k = \prod_{l=1}^{r} \left(h_l(t)^{k_l}\right),\ h_l(t-1) = h_l(\varsigma(t-1)). \end{cases} \tag{3}$$

Some illustrations are given here. For $r = 3$, $k = k_1 k_2 k_3 = 211 \in \mathcal{K}(4)$, $k' = k_1' k_2' k_3' = 101 \in \mathcal{K}(3)$, and $h_1(t) = 0.3, h_2(t) = 0.2, h_3(t) = 0.5$, we get $h(t)^k = 0.3^2 \times 0.2^1 \times 0.5^1 = 0.009$, $k + k' = 211 + 101 = 312$, $k - k' = 211 - 101 = 110$, and $k - \chi_3 = 211 - 001 = 210$.

Next, the section is concluded with the following Lemma and property about homogeneous matrix polynomials.

Lemma 1 (See [33]): For $\zeta \in \mathbb{R}^{n_1}, \Xi \in \mathbb{R}^{n_1 \times n_1}, S \in \mathbb{R}^{n_2 \times n_1}$, $\text{rank}(S) < n_1$, one has

$$\zeta^T \Xi \zeta < 0 \forall \zeta \neq 0, \quad \text{such that} \quad S\zeta = 0$$

if and only if $\exists Q \in \mathbb{R}^{n_1 \times n_2}$ satisfying that $\Xi + \text{He}(QS) < 0$.

Property 1: For an integer $d \in \mathbb{Z}_+$, a scalar α, and matrices X_k^m and V_k^{mj} for $k \in \mathcal{K}(d-1), m, j \in \{1, \ldots, r\}$ and $j \neq m$, we get two equalities as follows:

$$\left(h_m(t) - \alpha \sum_{\binom{j \in \{1,\ldots,r\}}{j \neq m}} h_j(t)\right) \sum_{k \in \mathcal{K}(d-1)} \left\{h(t)^k X_k^m\right\}$$

$$= \sum_{k'' \in \mathcal{K}(d)} \left\{ h(t)^{k''} \left(\sum_{\binom{l \in \{1,\ldots,r\}}{k'' - \chi_l \geq 0}} \Theta_{k''}^{ml} \right) \right\}$$

$$\sum_{\binom{j \in \{1,\ldots,r\}}{j \neq m}} \left\{ (h_m(t) - h_j(t)) \sum_{k \in \mathcal{K}(d-1)} \left\{ h(t)^k V_k^{mj} \right\} \right\}$$

$$= \sum_{k'' \in \mathcal{K}(d)} h(t)^{k''} \left(\sum_{\binom{l \in \{1,\ldots,r\}}{k'' - \chi_l \geq 0}} \Xi_{k''}^{ml} \right)$$

where

$$\Theta_{k''}^{ml} = \begin{cases} X_{k''-\chi_l}^m, & \text{for } l = m \\ -\alpha X_{k''-\chi_l}^m, & \text{for } l \neq m \end{cases}$$

$$\Xi_{k''}^{ml} = \begin{cases} \sum_{j \in \{1,\ldots,r\}, j \neq m} V_{k''-\chi_l}^{mj}, & \text{for } l = m \\ -V_{k''-\chi_l}^{ml}, & \text{for } l \neq m. \end{cases}$$

C. Alert Threshold Condition of Resilient Fuzzy Stabilization

In [32], the alert threshold condition of resilient fuzzy stabilization is expressed as a reasonable assumption, which will be encountered in the following stability analysis.

Assumption 1 (See [32]): For the underlying fuzzy system of (2), there must exist a scalar $\lambda \in (0, 1)$ such that the alert threshold condition is satisfied for all the sampling instants

$$\frac{x^T(t) \left(P_s(t-1)\right)^{-1} x(t)}{x^T(t) \left(P_s(t)\right)^{-1} x(t)} \geq \lambda \tag{4}$$

where two Lyapunov matrices are given as $P_s(t-1) = \sum_{k' \in \mathcal{K}(s)} \{h(t-1)^{k'} P_{k'}\}$ and $P_s(t) = \sum_{k' \in \mathcal{K}(s)} \{h(t)^{k'} P_{k'}\}$, respectively. $P_{k'} \in \mathbb{R}^{n_1 \times n_1}$, $s \in \mathbb{Z}_+$.

Remark 1: The difference between $x^T(t)(P_s(t-1))^{-1}x(t)$ and $x^T(t)(P_s(t))^{-1}x(t)$ is caused by $\Delta h(t) = h(t) - h(t-1)$. The alert threshold condition (4) may be violated for some large initial states with bigger values of "λ," especially for initial conditions far from the origin, and this indicates the necessity of a local analysis condition in Section IV. Similar to the general context where resilience in control systems regards keeping an acceptable level of performance when the system is subject to the uncertainty, the main feature of our resilient fuzzy stabilization is that the closed-loop system can remain asymptotically stable against the uncertain $\Delta h(t)$. Here, we define a real-time function $\lambda(t) = \frac{x^T(t)(P_s(t-1))^{-1}x(t)}{x^T(t)(P_s(t))^{-1}x(t)}$ and it is easy to check whether the previous assumption $\lambda(t) \geq \lambda$ holds in true at each sampling instant. It should be noted that the inequality (4) becomes much easy to be satisfied if λ is chosen as a small scalar, such as $\lambda = 0.25, 0.3$, and so on. Therefore, this short article aims at developing much relaxed resilient fuzzy stabilization while the alert threshold condition becomes more difficult to be violated than the existing one of [32].

III. Main Results

In this short article, the switching-type gain-scheduling control law is proposed while its working modes is denoted by $m\pm$ with $m \in \{1, \ldots, r\}$s

$$u(t) = Z_g^{m\pm}(t) \left(G_g^{m\pm}(t)\right)^{-1} x(t) \tag{5}$$

where we have

$$Z_g^{m\pm}(t) = \sum_{k\in\mathcal{K}(g)} \left\{h(t)^k Z_k^{m\pm}\right\}$$

$$G_g^{m\pm}(t) = \sum_{k\in\mathcal{K}(g)} \left\{h(t)^k G_k^{m\pm}\right\}.$$

And $g \in \mathbb{Z}_+$, $(Z_k^{m\pm} \in \mathbb{R}^{n_2\times n_1}$, $G_k^{m\pm} \in \mathbb{R}^{n_1\times n_1})$ belong to a pair of gain matrices to be determined. For the working modes $m+$, we have $h_m(t) = \max\{h_l(t), l \in \{1,\ldots,r\}\}$ and $h_m(t) \geq \alpha \sum_{j\in\{1,\ldots,r\},j\neq m} h_j(t)\}$ with $\alpha \geq 1$. For the working modes $m-$, we have $h_m(t) = \max\{h_l(t), l \in \{1,\ldots,r\}\}$ and $h_m(t) < \alpha \sum_{j\in\{1,\ldots,r\},j\neq m} h_j(t)\}$ with $\alpha \geq 1$.

Theorem 1: For two prescribed scalars $0 < \lambda < 1$ and $\alpha \geq 1$, and two prescribed integers $g, s \in \mathbb{Z}_+$, the underlying fuzzy system of (2) is thought to be asymptotically stable under the control of the switching-type gain-scheduling control law (5), if there exist symmetric matrices $P_{k'} \in \mathbb{R}^{n_1\times n_1}$ $(k' \in \mathcal{K}(s))$, gain matrices $Z_k^{m\pm} \in \mathbb{R}^{n_2\times n_1}$, $G_k^{m\pm} \in \mathbb{R}^{n_1\times n_1}$ $(k \in \mathcal{K}(g)$, and positive definite balanced matrices $X_k^m \in \mathbb{R}^{2n_1\times 2n_1}$ and negative definite balanced matrices $Y_k^m \in \mathbb{R}^{2n_1\times 2n_1}$ with $m \in \{1,\ldots,r\}$, $k \in \mathcal{K}(d-1)$; positive definite balanced matrices $V_k^{mj} \in \mathbb{R}^{2n_1\times 2n_1}$ with $m, j \in \{1,\ldots,r\}$ and $j \neq m$, $k \in \mathcal{K}(d-1)$; satisfying that the following LMIs of (6) can be ensured together:

$$\Upsilon_{k''}^{m\pm} < 0 \quad \forall k'' \in \mathcal{K}(d) \tag{6}$$

where we define $d = \max\{g+1, s\}$, and $\Upsilon_{k''}^{m\pm}$ is given as follow:

$$\Upsilon_{k''}^{m\pm} = \sum_{k'\in\mathcal{K}(s),k''\geq k'} \left\{\varphi_{k'k''}\begin{bmatrix}\frac{1}{\lambda}P_{k'} & *\\ 0 & -P_{k'}\end{bmatrix}\right\} + \sum_{\binom{k\in\mathcal{K}(g),j\in\{1,\ldots,r\}}{k''-k-\chi_j\geq 0}} \left\{\psi_{k'k''}^j \begin{bmatrix}-\mathrm{He}\left(G_k^{m\pm}\right) & *\\ A_jG_k^{m\pm}+B_jZ_k^{m\pm} & 0\end{bmatrix}\right\} + \sum_{\binom{l\in\{1,\ldots,r\}}{k''-\chi_l\geq 0}} \Gamma_{k''}^{m\pm l}$$

$\varphi_{k'k''} = \frac{(d-s)!}{\pi(k''-k')}$, $\psi_{k'k''}^j = \frac{(d-g-1)!}{\pi(k''-k-\chi_j)}$

$$\Gamma_{k''}^{m\pm l} = \begin{cases} X_{k''-\chi_l}^m, & \text{for mode } m+, \text{ and } l = m\\ -\alpha X_{k''-\chi_l}^m, & \text{for mode } m+, \text{ and } l \neq m\\ Y_{k''-\chi_l}^m + \sum_{j\in\{1,\ldots,r\},j\neq m} V_{k''-\chi g_l}^{mj} & \text{for mode } m-, \text{ and } l = m\\ -\alpha Y_{k''-\chi_l}^m - V_{k''-\chi_l}^{ml}, & \text{for mode } m-, \text{ and } l \neq m. \end{cases}$$

Proof: The applied Lyapunov function candidate of this short article is chosen as

$$V(x(t), \varsigma(t-1)) = x^T(t)\left(P_s(t-1)\right)^{-1}x(t) = x^T(t)\left(\sum_{k'\in\mathcal{K}(s)}\left\{h(t-1)^{k'}P_{k'}\right\}\right)^{-1}x(t). \tag{7}$$

At the current sampling instant, if Assumption 1 holds in true $(-x^T(t)P_s(t-1)^{-1}x(t) \leq -x^T(t)(\lambda P_s(t)^{-1})x(t))$, then the forward difference of the applied Lyapunov function candidate (7) is obtained

$$\Delta V(x(t), \varsigma(t-1)) = V(x(t+1), \varsigma(t)) - V(x(t), \varsigma(t-1))$$

$$= \zeta(t,t+1)^T \begin{pmatrix}-P_s(t-1)^{-1} & 0\\ 0 & P_s(t)^{-1}\end{pmatrix}\zeta(t,t+1)$$

$$\leq \zeta(t,t+1)^T \begin{pmatrix}-\lambda P_s(t)^{-1} & 0\\ 0 & P_s(t)^{-1}\end{pmatrix}\zeta(t,t+1) \tag{8}$$

where $\zeta(t,t+1) = \binom{x(t)}{x(t+1)}$, $P_s(t) = \sum_{k'\in\mathcal{K}(s)}\{h(t)^{k'}P_{k'}\}$.

Recalling (2) and (5), it is easy to obtain

$$\left(A_{\varsigma(t)} + B_{\varsigma(t)}Z_g^{m\pm}(t)\left(G_g^{m\pm}(t)\right)^{-1} \quad -I\right)\zeta(t,t+1) = 0. \tag{9}$$

Using Lemma 1 to (8)–(9), $\Delta V(x(t), \varsigma(t-1)) < 0$ can be guaranteed if there exists one matrix $Q \in \mathbb{R}^{2n_1\times n_1}$ such that

$$\mathrm{He}\left(Q\left(A_{\varsigma(t)} + B_{\varsigma(t)}Z_g^{m\pm}(t)\left(G_g^{m\pm}(t)\right)^{-1} \quad I\right)\right) + \begin{pmatrix}-\lambda P_s(t)^{-1} & 0\\ 0 & P_s(t)^{-1}\end{pmatrix} < 0 \tag{10}$$

where $Q = \binom{0}{P_s(t)^{-1}}$ for this article.

Pre- and postmultiplying the abovementioned inequality (10) with $\begin{pmatrix}G_g^{m\pm}(t) & 0\\ 0 & P_s(t)\end{pmatrix}^T$ and its transpose, an inequality for guaranteeing (10) is obtained by considering $\frac{1}{\lambda}P_s(t) - \mathrm{He}(G_g^{m\pm}(t)) \geq -(G_g^{m\pm}(t))^T(\lambda P_s(t)^{-1})G_g^{m\pm}(t)$:

$$\begin{pmatrix}\frac{1}{\lambda}P_s(t) - \mathrm{He}(G_g^{m\pm}(t)) & *\\ A_{\varsigma(t)}G_g^{m\pm}(t) + B_{\varsigma(t)}Z_g^{m\pm}(t) & -P_s(t)\end{pmatrix} < 0. \tag{11}$$

Further, all the possible modes $m\pm$ can be divided into two categories, i.e., the possible modes $m+$ and the other possible modes $m-$. When one of the possible modes $m+$ is enabled, we get

$$\left(h_m(t) - \alpha \sum_{\binom{j\{1,\ldots,r\}}{j\neq m}} h_j(t)\right)\sum_{k\in\mathcal{K}(d-1)} h(t)^k X_k^m \geq 0 \tag{12}$$

where it implies that $\mathrm{Left}(12) \geq 0$ for $m+$.

When one of the possible modes $m-$ is enabled, we get

$$\left(h_m(t) - \alpha \sum_{\binom{j\{1,\ldots,r\}}{j\neq m}} h_j(t)\right)\sum_{k\in\mathcal{K}(d-1)} h(t)^k Y_k^m \geq 0 \tag{13}$$

$$\sum_{\binom{j\{1,\ldots,r\}}{j\neq m}}\left\{(h_m(t) - h_j(t))\sum_{k\in\mathcal{K}(d-1)} h(t)^k V_k^{mj}\right\} \geq 0 \tag{14}$$

where it implies that $\mathrm{Left}(13) + \mathrm{Left}(14) \geq 0$ for $m-$.

Combining (12)–(14), the inequality (11) is guaranteed by the following inequality:

$$\mathrm{Left}(11) + \Gamma^{m\pm} < 0 \tag{15}$$

while $\Gamma^{m\pm} = \begin{cases}\mathrm{Left}(12) \geq 0, & \text{for } m+\\ \mathrm{Left}(13) + \mathrm{Left}(14) \geq 0, & \text{for } m-.\end{cases}$

Moreover, we also have

$$\mathrm{Left}(11) = \sum_{k'\in\mathcal{K}(s),k''\geq k'} \left\{\varphi_{k'k''}\begin{bmatrix}\frac{1}{\lambda}P_{k'} & *\\ 0 & -P_{k'}\end{bmatrix}\right\} + \sum_{\binom{k\in\mathcal{K}(g),j\in\{1,\ldots,r\}}{k''-k-\chi_j\geq 0}} \left\{\psi_{k'k''}^j \begin{bmatrix}-\mathrm{He}\left(G_k^{m\pm}\right) & *\\ A_jG_k^{m\pm}+B_jZ_k^{m\pm} & 0\end{bmatrix}\right\}$$

where $\varphi_{k'k''}$ and $\psi_{k'k''}^j$ are given in (6).

Thus, an important equality is acquired by using Property 1

$$\text{Left}(15) = \sum_{k'' \in \mathcal{K}(d)} \left\{ h(t)^{k''} \Upsilon_{k''}^{m\pm} \right\} \tag{16}$$

where $\Upsilon_{k''}^{m\pm}$ is given in (6).

Therefore, $\text{Left}(15) < 0$ is fulfilled by $\Upsilon_{k''}^{m\pm} < 0 \forall k'' \in \mathcal{K}(d),\ m \in \{1, \ldots, r\}$. That is to say, the considered system of (2) with the gain-scheduling control law (5) becomes asymptotically stable if all the LMIs of (6) are guaranteed. ■

Remark 2: If the switching-type gain-scheduling control law (5) degenerates into the nonswitching control law (i.e., $Z_g^{1+} = Z_g^{1-} = \cdots = Z_g^{r+} = Z_g^{r-}$ and $G_g^{1+} = G_g^{1-} = \cdots = G_g^{r+} = G_g^{r-}$) and the key term $\sum_{\binom{l \in \{1,\ldots,r\}}{k''-\chi_l \geq 0}} \Gamma_{k''}^{m\pm l}$ is removed from $\Upsilon_{k''}^{m\pm}$, then the proposed condition of (6) is reduced to the main result given in [32] as a special case. In other words, the proposed strategy reduces the conservativeness of the alert threshold condition of [32]. Furthermore, the inequalities in (6) can be numerically solved considering the dependence on the modes "$m\pm$." For example, if we have $r = 3$ and $d = 2$, then (6) produces the following inequalities: $\Upsilon_{k''}^{1+} < 0$, $\Upsilon_{k''}^{1-} < 0$, $\Upsilon_{k''}^{2+} < 0$, $\Upsilon_{k''}^{2-} < 0$, $\Upsilon_{k''}^{3+} < 0$, and $\Upsilon_{k''}^{3-} < 0$, with $k'' \in \mathcal{K}(2) = \{200, 020, 002, 110, 101, 011\}$, respectively.

Remark 3: Two prescribed scalars $\alpha \geq 1$ and $0 < \lambda < 1$ are both important for the conservative level of our obtained fuzzy stabilization criterion (6). A systematic methodology to select these parameters of "α" and "λ" are given as follows: As for α, we test a finite number of alternative values, such as $\alpha \in \{1, 1.5, 2, 2.5, 3, 4\}$, and then, the variation trend of the conservative level along different α can be obtained so that the best value of α can be chosen in accordance with this variation trend. As for λ, although less conservative results can be obtained with the bigger values of λ, it should be pointed out that there is a tradeoff between the region of attraction of the closed-loop equilibrium (where Assumption 1 is always guaranteed) and the value of λ. Therefore, it is reasonable to choose the value of λ as small as possible while all the involved LMIs of (6) must be feasible as a prerequisite. Moreover, the selection rules of α and λ will be shown in Section IV for details.

IV. Numerical Simulation

Example: Recalling the benchmark example that is given in [21]

$$\begin{cases} x_1(t+1) = x_1(t) - x_1(t)x_2(t) + (5 + x_1(t))u(t) \\ x_2(t+1) = -x_1(t) - 0.5x_2(t) + 2x_1(t)u(t). \end{cases} \tag{17}$$

Borrowed from [21], two NFWFs are given by: $h_1(\varsigma(t)) = \mathcal{M}_1^1(x_1(t)) = (b_{\max} + x_1(t))/2b_{\max}$ and $h_2(\varsigma(t)) = \mathcal{M}_1^2(x_1(t)) = (b_{\max} - x_1(t))/2b_{\max}$, respectively. It should be noted that the conservative level of different methods can be compared with their corresponding $b_{\max}$ in this case study. The investigated plant is modeled by the T–S fuzzy system (2) with a set of parameter matrices

$$A_1 = \begin{bmatrix} 1 & -b_{\max} \\ -1 & -0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 5 + b_{\max} \\ 2b_{\max} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & b_{\max} \\ -1 & -0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 5 - b_{\max} \\ -2b_{\max} \end{bmatrix}.$$

First, choosing the same pair of degrees for gain matrices as those given in [31] and [32] (i.e., $g = 2$ and $s = 2$ for Theorem 1), the obtained $b_{\max}$ of Theorem 1 for different $\alpha \in \{1.5, 2, 2.5, 3, 4\}$ and different $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$ have been given in Table I. From each line of Table I, we can find that there exists a wave crest in the variation trend of $b_{\max}$ when α is fixed, and thus, the best value of α for this case study can be chosen as $\alpha = 2$ for $\lambda \in \{0.2, 0.3, 0.4\}$ or $\alpha = 2.5$ for $\lambda \in \{0.5, 0.6\}$. From each column of Table I, we can find that a bigger λ does produce a bigger $b_{\max}$. However, a bigger λ also means that the violation of Assumption 1 is much easier and this is not what we expected. Therefore, it is much reasonable to choose the smaller λ while the corresponding conservative level $b_{\max}$ should also be satisfactory. For this case study, we obtain $b_{\max} = 1.896$ even if λ is chosen as the smallest value 0.2, and this means the violation of Assumption 1 is very difficult. Recalling the fact that all the methods given in [29]–[31] fail to give any feasible solution as $b_{\max} > 1.830$, it can be said that resilient fuzzy stabilization of this study has proposed an alternative for obtaining significantly increased value of $b_{\max}$ than those of existing conventional fuzzy stabilization results, e.g., [29]–[31].

Second, the result of this short article is compared with the recent result given in [32]. In order to give comprehensive and fair comparisons, the conservative level (characterized by $b_{\max}$) is considered in this case study, i.e., the comparisons of $b_{\max}$ of [32] and Theorem 1 for different $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$ as $\alpha = 2$ have been given in Table II. Seen from each column of Table II, we can find that the obtained $b_{\max}$ of ours is evidently bigger than the obtained $b_{\max}$ of [32] since the increase rates have also been given in Table II. For visual display, these results have also been displayed in Fig. 1. From Fig. 1, the dark blue solid line (ours) is always beyond the red line $b_{\max} = 1.830$ (which is the limit value obtained by conventional fuzzy stabilization of [31]) as $\lambda \geq 0.2$, and the blue dotted line (result of [32]) is beyond the red line $b_{\max} = 1.830$ only as $\lambda \geq 0.35$. Moreover, the dark blue solid line (ours) is always beyond the blue dotted line (result of [32]). In other words, the conservatism

TABLE I
Obtained $b_{\max}$ of Theorem 1 for Different $\alpha \in \{1.5, 2, 2.5, 3, 4\}$ and Different $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$

λ/α	1.5	2	2.5	3	4
0.2	1.889	1.896	1.886	1.880	1.873
0.3	2.243	2.293	2.273	2.257	2.237
0.4	2.643	2.729	2.750	2.714	2.662
0.5	2.902	2.959	2.987	2.953	2.914
0.6	3.002	3.046	3.070	3.061	3.015

TABLE II
Comparisons of $b_{\max}$ of [32] and Theorem 1 for Different $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$ as $\alpha = 2$

λ	0.2	0.3	0.4	0.5	0.6
[32]	1.377	1.692	1.968	2.180	2.366
Ours	1.896	2.293	2.729	2.959	3.046
increase rate	37.7%	35.5%	38.7%	35.7%	28.7%

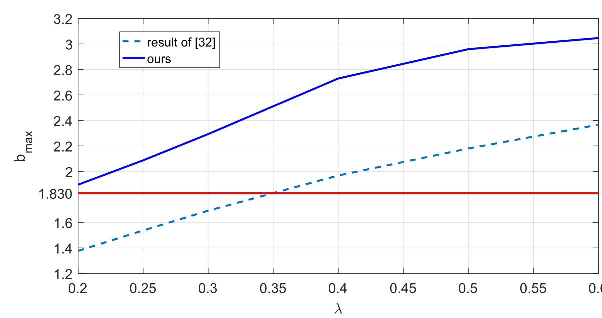

Fig. 1. Variation trend of $b_{\max}$ along different λ for [32] (blue dotted line) and ours (dark blue solid line) while $b_{\max} = 1.830$ is the limit value obtained by conventional fuzzy stabilization of [31].

TABLE III
COMPARISONS OF COMPUTATIONAL BURDEN BETWEEN [32] AND THEOREM 1

Methods	N_L	N_D	$\log_{10}\left(N_D^3 N_L\right)$
[32]	20	36	5.9699
Ours	136	261	9.3835

of resilient fuzzy stabilization has been further reduced even if the alert threshold condition becomes more difficult to be violated than the recent result of [32]. Based on the previous comparisons, it is evident that the proposed method is able to provide less conservative results than other related works. However, it comes at the price of introducing slack variables, thus increasing the computational burden. In this sense, we have indicated the number of decision variables N_D, the number of LMI rows N_L, and the well-known indicator of $\log_{10}(N_D^3 N_L)$ in Table III, respectively. Third, if we set $b_{\max} = 3.046$, we notice from Table II that all the results given in [32] cannot provide any feasible solution for this case study. In this case, using Theorem 1 of this short article with $g = s = 2$, $\lambda = 0.6$, and $\alpha = 2$, four groups of gain matrices $(G_k^{m\pm}, Z_k^{m\pm},\ m \in \{1,2\},\ k \in \{20,11,02\})$ are obtained by solving all the LMIs of (6).

First working mode: $m\pm = 1+$

$$G_{20}^{1+} = \begin{bmatrix} 10.2208 & 12.9013 \\ 8.9267 & 12.0409 \end{bmatrix}$$

$$G_{11}^{1+} = \begin{bmatrix} -15.0711 & -29.4820 \\ 6.0751 & 12.1188 \end{bmatrix}$$

$$G_{02}^{1+} = \begin{bmatrix} 17.7364 & 1.1546 \\ -3.2048 & 0.4516 \end{bmatrix}$$

$$Z_{20}^{1+} = \begin{bmatrix} 1.5048 & 2.6519 \end{bmatrix}$$

$$Z_{11}^{1+} = \begin{bmatrix} -1.1047 & 1.7179 \end{bmatrix}$$

$$Z_{02}^{1+} = \begin{bmatrix} -2.4684 & -1.0393 \end{bmatrix}.$$

Second working mode: $m\pm = 1-$

$$G_{20}^{1-} = \begin{bmatrix} 10.4620 & 15.8736 \\ 8.3146 & 14.8648 \end{bmatrix}$$

$$G_{11}^{1-} = \begin{bmatrix} -14.4313 & -29.7409 \\ 4.4133 & 16.4477 \end{bmatrix}$$

$$G_{02}^{1-} = \begin{bmatrix} 20.2380 & -1.5014 \\ -5.3768 & 4.6443 \end{bmatrix}$$

$$Z_{20}^{1-} = \begin{bmatrix} 1.0956 & 3.2156 \end{bmatrix}$$

$$Z_{11}^{1-} = \begin{bmatrix} -1.3807 & 1.7824 \end{bmatrix}$$

$$Z_{02}^{1-} = \begin{bmatrix} -2.6491 & -1.4283 \end{bmatrix}.$$

Third working mode: $m\pm = 2+$

$$G_{20}^{2+} = \begin{bmatrix} 13.8683 & -7.0794 \\ 15.8029 & 7.8112 \end{bmatrix}$$

$$G_{11}^{2+} = \begin{bmatrix} -24.4892 & -18.3359 \\ 7.2233 & 8.8727 \end{bmatrix}$$

$$G_{02}^{2+} = \begin{bmatrix} 22.9239 & 2.2772 \\ -9.4237 & 3.6844 \end{bmatrix}$$

$$Z_{20}^{2+} = \begin{bmatrix} 1.8085 & 2.4555 \end{bmatrix}$$

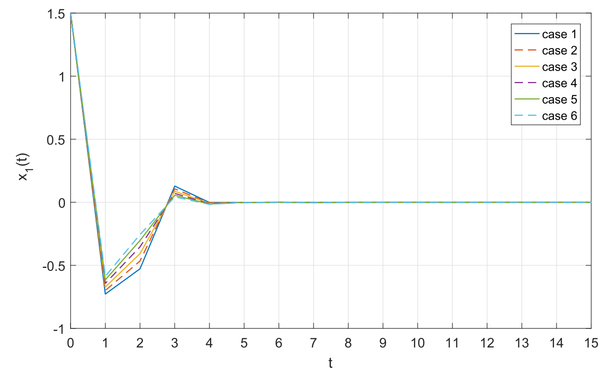

Fig. 2. Responses of $x_1(t)$ for cases 1–6.

$$Z_{11}^{2+} = \begin{bmatrix} -1.3696 & 1.0875 \end{bmatrix}$$

$$Z_{02}^{2+} = \begin{bmatrix} -2.9534 & -0.7140 \end{bmatrix}.$$

Fourth working mode: $m\pm = 2-$

$$G_{20}^{2-} = \begin{bmatrix} 14.4285 & 18.3558 \\ 7.5940 & 17.0272 \end{bmatrix}$$

$$G_{11}^{2-} = \begin{bmatrix} -16.9036 & -32.5383 \\ 2.3765 & 16.4926 \end{bmatrix}$$

$$G_{02}^{2-} = \begin{bmatrix} 21.5168 & 4.2335 \\ -7.5482 & 3.2641 \end{bmatrix}$$

$$Z_{20}^{2-} = \begin{bmatrix} 0.4269 & 3.1817 \end{bmatrix}$$

$$Z_{11}^{2-} = \begin{bmatrix} -1.4833 & 1.2145 \end{bmatrix}$$

$$Z_{02}^{2-} = \begin{bmatrix} -2.4049 & -2.5051 \end{bmatrix}.$$

Furthermore, the obtained three Lyapunov matrices P_{20}, P_{11}, and P_{02} are given as follows:

$$P_{20} = \begin{bmatrix} 10.9700 & 12.4465 \\ 12.4465 & 14.1220 \end{bmatrix}$$

$$P_{11} = \begin{bmatrix} -23.9447 & -14.9452 \\ -14.9452 & 13.5919 \end{bmatrix}$$

$$P_{02} = \begin{bmatrix} 20.3941 & 0.3871 \\ 0.3871 & 0.0092 \end{bmatrix}.$$

Without loss of generality, the following six different initial conditions of the underlying plant are considered in the same place with $x_1(0) = 1.5$:

Case 1: $x_2(0) = 1.0$;
Case 2: $x_2(0) = 2.0$;
Case 3: $x_2(0) = 3.0$;
Case 4: $x_2(0) = 4.0$;
Case 5: $x_2(0) = 5.0$;
Case 6: $x_2(0) = 6.0$.

With the gain-scheduling control law (5), the response curves of $x_1(t)$, $x_2(t)$, and $\lambda(t) = \frac{x^T(t)(P_s(t-1))^{-1}x(t)}{x^T(t)(P_s(t))^{-1}x(t)}$ with six different initial conditions are displayed in Figs. 2–4, respectively. It can be found from Figs. 2 and 3 that all the obtained $x_1(t)$ and $x_2(t)$ remain asymptotically stable. More importantly, all the obtained $\mu(t)$ are beyond the red line of $\lambda = 0.6$ in Fig. 4 (i.e., the establishment of the underlying Assumption 1 has always been guaranteed in the whole process of real-time control).

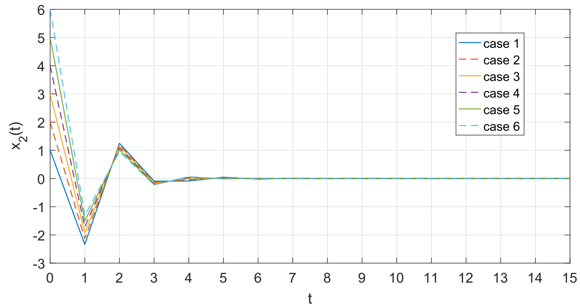

Fig. 3. Responses of $x_2(t)$ for cases 1–6.

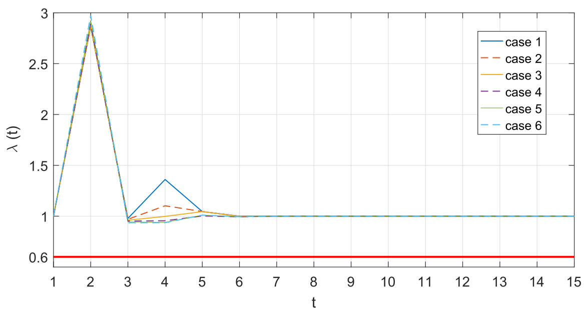

Fig. 4. Responses of $\lambda(t)$ for cases 1–6.

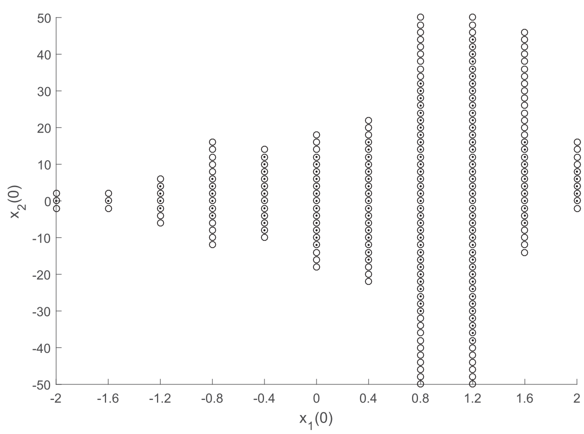

Fig. 5. Stability regions for [32] (●) and Theorem 1 (◯).

Further, for $b_{\max} = 2.0$ and $g = s = 2$, the smallest λ obtained by the method of [32] is $\lambda = 0.42$ and the smallest λ obtained by Theorem 1 is $\lambda = 0.23$. Fig. 5 gives the set of initial conditions leading to asymptotically stable responses while $-b_{\max} \leq x_1(t) \leq b_{\max}$ and $\lambda(t) \geq \lambda$ are both guaranteed at all sampling instants. It can be noticed that the stability region for Theorem 1 (◯) encompasses the stability region for [32] (●).

Finally, with the purpose of testing the generality of our proposed solution, it should be executed on other T–S fuzzy alternatives for the same nonlinear plant. Here, we employ the sum normalization non-negativeness (SNNN)-type antecedents and the powerful tensor product model transformation given in [18] and [34]–[36] in order to produce different fuzzy rules of the same nonlinear plant (17) with $b_{\max} = 3.046$.

With the help of the TP MATLAB toolbox [37], the underlying nonlinear plant (17) with $b_{\max} = 3.046$ is converted to the fuzzy model in terms of (2) with two SNNN-type antecedent NFWFs, as shown in

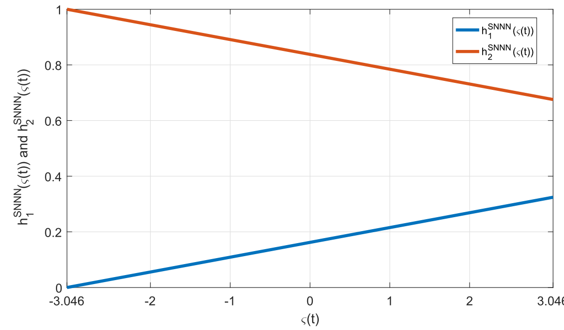

Fig. 6. SNNN-type antecedent NFWFs.

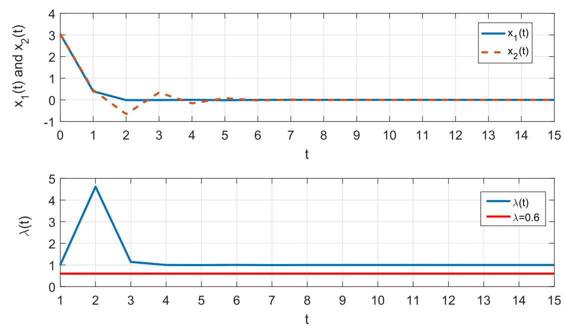

Fig. 7. Responses of $x_1(t)$, $x_2(t)$, and $\lambda(t)$.

Fig. 6, and its new SNNN-type parameter matrices are produced as follows:

$$A_1^{\text{SNNN}} = \begin{bmatrix} 1.0000 & -15.7325 \\ -1.0000 & -0.5000 \end{bmatrix}$$

$$B_1^{\text{SNNN}} = \begin{bmatrix} 20.7325 \\ 31.4650 \end{bmatrix}$$

$$A_2^{\text{SNNN}} = \begin{bmatrix} 1.0000 & 3.0460 \\ -1.0000 & -0.5000 \end{bmatrix}$$

$$B_2^{\text{SNNN}} = \begin{bmatrix} 1.9540 \\ -6.0920 \end{bmatrix}.$$

Then, applying the gain-scheduling control law (5) [whose four groups of gain matrices are obtained by solving all the LMIs of (6) with the above SNNN-type parameter matrices] to the fuzzy model in terms of (2) with two SNNN-type antecedent NFWFs the response curves of $x_1(t)$, $x_2(t)$, and $\lambda(t)$ are displayed in Fig. 7 while the initial conditions are chosen as $x_1(0) = x_2(0) = b_{\max}$. It can be found from Fig. 7 that all the obtained $x_1(t)$ and $x_2(t)$ are asymptotically stable. What is more, the obtained $\mu(t)$ is also beyond the red line of $\lambda = 0.6$ in Fig. 7 (i.e., the establishment of the underlying Assumption 1 has always been guaranteed in the whole process of real-time control). Therefore, the generality of our proposed solution behaves well in this case study.

V. Conclusion

In order to make the alert threshold condition of resilient fuzzy stabilization more easily to be guaranteed, this short article has employed the switching-type gain-scheduling control law so that the real-time information, which is specific to the current sampling instant, can be integrated into resilient fuzzy stabilization. Furthermore, a new kind of time-variant balanced matrix has been introduced for adjusting positive/negative terms of different monomials in a more flexible way. Therefore, the conservatism of resilient fuzzy stabilization can be further reduced even if the alert threshold condition becomes more difficult to be violated than the recent result of [32]. Finally, the advantage of the developed method has been tested and validated via related comparisons on the benchmark example. In the future research, the topic of proposing much more relaxed stabilization criterion for T–S fuzzy systems with malicious attacks (i.e., the information of $x(t)$ may be tampered or even lost at some sampling instants) belongs to one challenging but valuable problem.

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